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On the numerical computation of the optimum profile in Stokes flow

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The body of given volume with the smallest drag in Stokes flow is obtained by making use of theoretical results due to Pironneau. A suitable family of solutions of the Stokes equations is used and the no-slip condition is expressed numerically by a technique of quadratic minimization. We find that the drag on this optimal body is 0·95425 times the drag on the sphere of equal volume.

In a recent paper, Pironneau (1973) found that the body of given volume which has the smallest drag in a uniform flow at low Reynolds number must be such that the vorticity is the same at every point of its surface. From this, he deduced that, if the solution is unique, it must be axisymmetric, it must have a centre of symmetry and it must have conical front and rear ends of angle 120°. From those results we shall compute numerically the optimal shape.

We construct the hydrodynamical field around a given profile P_0 by developing the stream function in terms of prolate spheroidal harmonics, completed by adding two terms which represent the Stokes flow around the conical ends (Bourot 1974). The remaining parameters are adjusted so as to minimize the mean square of the fluid speed on the boundary. This technique has been previously described by Bourot (1969).

Now all the hydrodynamical quantities can be computed. Let C_f be the ratio of the drag on P_0 to the drag on the sphere of equal volume; let ζ be the vorticity on P_0. Knowing these quantities enables us to perform successive alterations in order to obtain a sequence of profiles $P_1, P_2, \ldots$, with decreasing C_f. For this purpose we could apply the algorithm suggested by Pironneau; however, the technique used makes it easier to operate with a family of profiles depending on a finite number of parameters and to optimize these parameters with respect to C_f. Simultaneously, in accordance with Pironneau's result, we checked that ζ was converging towards a constant. The profile obtained in this way was already satisfactory but the computations could be refined further by rectifying the profile according to the local fluctuations in the vorticity.

The profile P_f that we reached is well represented by the equation

$$\frac{r(\theta)}{\lambda} = 2 - \frac{2}{\sqrt{3}}\sin\theta + \sum_{n=1}^{N} B_n \sin^{n+1}\theta,$$

where r denotes the distance from the centre, θ the colatitude and λ a scaling

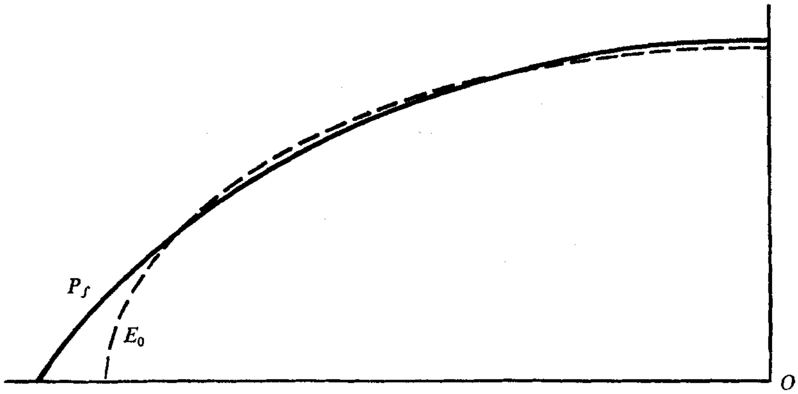

FIGURE 1

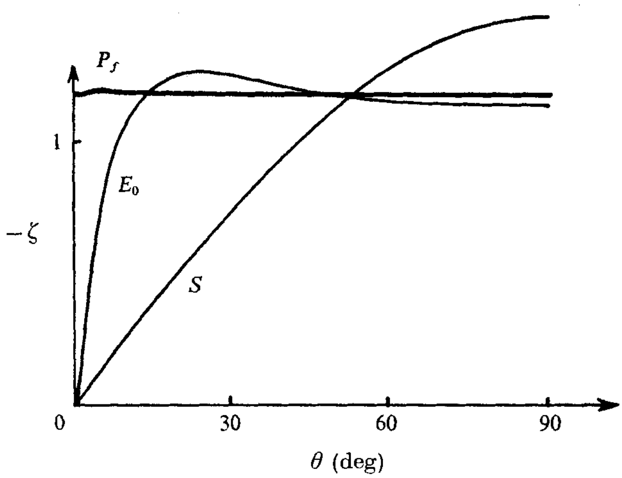

FIGURE 2

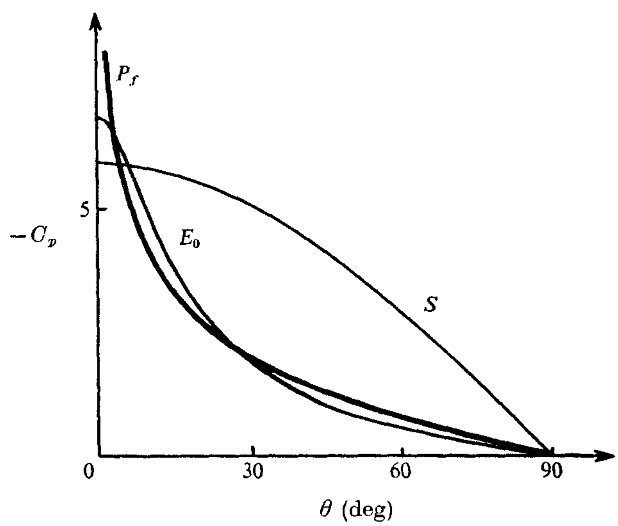

FIGURE 3

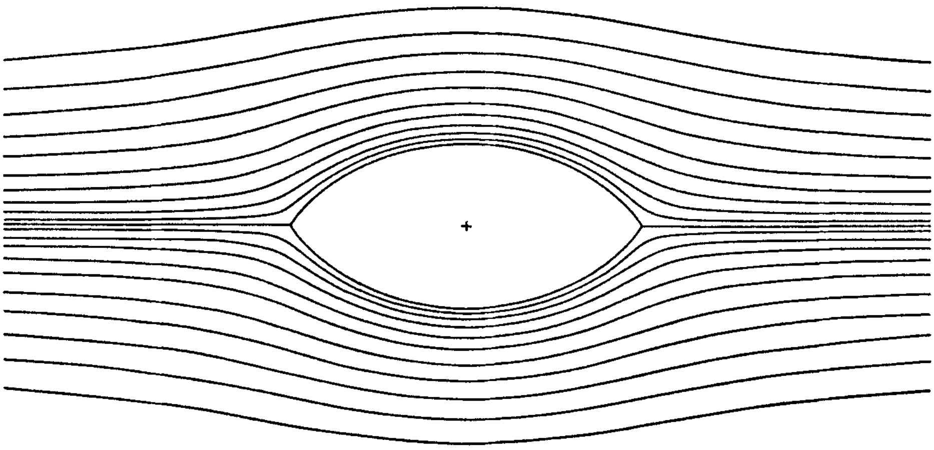

FIGURE 4

coefficient which makes its volume equal to that of the sphere S of unit radius; the values of the parameters are

$$N = 10, \quad \lambda = 0{\cdot}8588,$$
$$B_1 = -1{\cdot}847, \quad B_2 = 6{\cdot}367, \quad B_3 = -7{\cdot}212, \quad B_4 = -1{\cdot}964, \quad B_5 = 9{\cdot}365,$$
$$B_6 = 4{\cdot}137, \quad B_7 = -22{\cdot}31, \quad B_8 = 18{\cdot}32, \quad B_9 = -4{\cdot}502, \quad B_{10} = -0{\cdot}2446.$$

This profile is shown on figure 1, compared with the optimal prolate spheroid E_0 of equal volume. The drag coefficient is $C_f = 0{\cdot}95425$. Let us recall that for E_0 this coefficient is 0·95551. On figure 2 the vorticity distribution along P_f is drawn, compared with the distributions corresponding to E_0 and S. We see that P_f satisfies the optimality condition of Pironneau to a good accuracy.

On figure 3 we have drawn the distribution of the pressure coefficient

$$C_p = \frac{Re}{\frac{1}{2}\rho V_\infty^2}(p - p_\infty),$$

where Re is the Reynolds number based on the diameter of the equatorial circle.

Finally, figure 4 shows the streamlines around P_f.

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